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Statistical Modeling of Recent Changes in Extreme Rainfall in Taiwan

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Abstract: This paper has two primary purposes. First, we fit the annual maximum daily rainfall data for 6 rainfall stations, both with stationary and non-stationary generalized extreme value (GEV) distributions for the periods 1911-2010 and 1960-2010 in Taiwan, and detect the changes between the two phases for extreme rainfall. The non-stationary model means that the location parameter in the GEV distribution is a linear function of time to detect temporal trends in maximum rainfall. Second, we compute the future behavior of stationary models for the return levels of 10, 20, 50 and 100-years based on the period 1960-2010. In addition, the 95% confidence intervals of the return levels are provided. This is the first investigation to use generalized extreme value distributions to model extreme rainfall in Taiwan.

Keywords: Generalized extreme value, Extreme rainfall, Return level, Statistical modelling.

JEL Classifications: Q54, Q51, Q57

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1. Introduction.

Extreme rainfall is one of the main causes of natural disasters, especially in flood hazards worldwide. Not surprisingly, considerable attention has been paid in recent years to the modelling of extreme rainfall to help prevent flooding hazards, and for analysing water-related structures, agriculture, and monitoring climate changes.

Taiwan is a small island in East-Asia. An average of 3.5 typhoons strike Taiwan each year, often in summer and autumn, and cause significant damage, especially in highly concentrated population and property areas. In the past few years, the Kalmaegi and Sonlaku Typhoons in 2008 and Morakot Typhoon in 2009 have brought to Taiwan many casualties and serious injuries. Morakot Typhoon struck Taiwan from 7-9 August 2009 with abundant rainfall, reaching 2,777mm, and surpassing the historical record of Typhoon Herb, which had brought rainfall of 1,736mm ([1];[2]). The extremely heavy rainfall triggered severe flooding (the worst in the past 50 years) and enormous mudslides throughout southern Taiwan, leading to around 700 deaths and roughly NT$110 billion in property damage ([3]).

More typhoons seem to have affected Taiwan after 1990 than between 1961-1989 (see [5]), and increased sharply around 2000 (see [6]). Such an indication of increasing frequency and intensity of rainfall means that Taiwan will face a higher probability of huge damages from extreme rainfall in the future. Thus, understanding the patterns of extreme rainfall and their future behaviour is of increasing importance to policy makers in Taiwan.

Several published papers have analysed extreme rainfall using the generalized extreme value (GEV) distribution in different parts of the world, including Canada ([7, 8]); Greece [9]; India [10, 11]; Italy [12, 13, 14]; Malaysia [15]; New Zealand [16]; China [17]; Korea [18, 19], and West Central Florida [20]. These findings highlight the urgency to model extreme rainfall using the GEV distribution. However, there would seem to have been little or no published research that has attempted to detect extreme rainfall by using GEV. Therefore, this paper would seem to be the first application of the GEV distribution for extreme rainfall in Taiwan. We believe that such an analysis can provide a useful reference for climate change in the research of extreme rainfall.

2. Data.

The data consist of daily rainfall records for the years 1911-2010, which were provided by the Central Weather Bureau, Taiwan. Table 1 shows a statistical description of the six rainfall stations, including the station number, the longitude, the latitude, and a statistical summary of the data.

3. Methodology.

The generalized extreme value (GEV) distribution is based on the Gumbel, Fréchet and the Weibull distributions. It was developed by Jenkinson (1955), who combined the above three distributions (see Hosking et al., 1985, Galambos, 1987). The cumulative distribution function (cdf) of the GEV distribution is given as:

$$\text{Model GEV}_0\text{:}\quad F(x) = \exp\left\{-(1+\xi(x-\mu)/\sigma)^{-1/\xi}\right\}, \qquad 1+\xi(x-\mu)/\sigma > 0 \tag{1}$$

where μ, $\sigma > 0$ and ξ are the location, scale and shape parameters, respectively. The case of $\xi = 0$ in equation (1) is defined as the Gumbel distribution:

$$\text{Model Gum}_0\text{:}\quad F(x) = \exp\left\{-\exp\left(-(x-\mu)/\sigma\right)\right\}, \qquad -\infty < x < \infty \tag{2}$$

and the sub-families defined by $\xi > 0$ and $\xi < 0$ correspond to the Fréchet family and the Weibull family, respectively.

The maximum likelihood method was used to estimate (1) and (2) for these data, and the maximization was performed using a quasi-Newton iterative algorithm. Assuming independence of the data, the likelihood function is given as the product of the assumed densities for the observations $x_1, x_2, ..., n$. For the GEV$_0$ model, we have:

$$L(\mu,\sigma,\xi) = \frac{1}{\sigma^n}\prod_{i=1}^{n}\left(1+\xi(x_i-u)/\sigma\right)^{-(1/\xi+1)} \times \exp\left\{-\sum_{i=1}^{n}\left(1+\xi\frac{x_i-\mu}{\sigma}\right)^{-1/\xi}\right\} \tag{3}$$

provided that

$$1+\xi\frac{x_i-\mu}{\sigma}>0,\ \ i=1,\dots,n$$

The likelihood ratio test may be used as a standard way of determining the best fitting models. In order to investigate the existence of a deterministic trend in extreme rainfall over time, we consider the following variations of Models GEV_0 and Gum_0[1]:

Model GEV_1: $\mu=a+b$ (Year-t0+1), σ = constant, ξ = constant (4)

a four-parameter model with μ allowed to vary linearly with respect to time, and "constant" means that the parameter is not time dependent but is to be estimated:

Model Gum1: $\mu=a+b$ (Year-t0+1), σ = constant, ξ = 0, (5)

After the best models for the data have been determined, the next step is to derive the return levels for rainfall. The T-year return level, x_T , is the level exceeded on average only once every T years. If Model GEV_0 is assumed, then inverting $F(x_T)=1-1/T$ leads to the expression:

$$x_T=\mu-\frac{\sigma}{\xi}\left\{1-\left[-\log(1-1/T)\right]^{-\xi}\right\} \tag{6}$$

If Model Gum_0 is assumed, then the corresponding expression is given as:

$$x_T=\mu-\sigma\log\{-\log(1-1/T)\} \tag{7}$$

[1] We follow the same statistical methods as Nadarajah (2005), Feng et al. (2007), and Park et al. (2010). Consequently, some descriptions throughout this paper are the same as the above.

On substituting $\hat{\mu}, \hat{\sigma}$ and $\hat{\xi}$ into equations (6)-(7), we have the maximum likelihood estimates of the respective return levels.

4. Estimated Results.

All the empirical results, including the estimates and their corresponding standard errors (SE), are given in Table 2. It is evident that the Taipei, Taichung, Tainan and Taitung stations have consistent distributions for the two phases. The non-stationary Gumbel distribution is suitable for the Taitung station, while the stationary Gumel distribution is applicable for the Taipei, Taichung, Tainan stations for the two phases. The Hengchun station does not have a consistent distribution for the two periods. However, the Hualien station has a non-stationary GEV distribution for the period 1910-2010, but has a stationary GEV distribution for the period 1960-2010.

The Taipei and Tainan stations have a greater location parameter for the period 1960-2010 than for the period 1910-2010, showing a slight increase in extreme rainfall for the period 1960-2010. Similarly, the Taichung station has a slight decrease in extreme rainfall for the period 1960-2010. It is clear that the empirical results indicate extreme rainfall for the Taitung and Hualien stations have had a manifest increase during the period 1960-2010.

Table 3 gives the estimates of the return level, x_T, corresponding to 10, 20, 50, 100 years for the locations, where the best fitting and stationary model for the period 1960-2010 was chosen. The 95% confidence intervals for these return levels are also provided. The empirical results indicate that, given 50-year return level (for the year 2060), the return levels of daily extreme rainfall of the Taipei, Taichung, Tainan and Hualien stations were all greater than 350mm, reaching a warning line of extremely torrential rainfall, as defined by the CWB in Taiwan[2].

Figure 1-3 show the contour maps of the return levels for 10, 20 and 50-years, respectively. These contour maps can assist in determining changes in extreme rainfall in a simple manner.

5. Conclusion.

This is the first investigation to use the generalized extreme value to model extreme rainfall in Taiwan. We estimated extreme rainfall using stationary and non-stationary models for both 1910-2010 and 1960-2010 to detect changes in extreme rainfall for these two periods. The empirical results indicated that the stationary Gumbel distribution was suitable for the Taipei, Taichung, and Tainan stations, while the non-stationary Gumbel distribution was suitable for the Taitung station.

[2] When the daily rainfall is greater than 50, 130, 200 and 350 mm, the rainfall will accord with the definitions of heavy rainfall, extremely heavy rainfall, torrential rainfall, and extremely torrential rainfall, respectively.

Among these stations, the Taipei and Tainan stations have a slight increase in extreme rainfall, while the Taitung station shows a manifest increase in extreme rainfall for the period 1961-2010. Given 50-year return level (for the year 2060), the return levels of daily extreme rainfall of the Taipei, Taichung, Tainan and Hualien stations were all greater than 350mm, reaching a warning line of extremely torrential rainfall, as defined by the CWB in Taiwan.

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Table 1.
Descriptive statistics of data set (unit: mm).

Location	Station No.	Longitude	Latitude	Min	Max	Media	Mean
Taipei	46692	121°51'	25°04'	20.6	425.2	145.9	163.5
Taichung	46749	120°68'	24°15'	25.8	660.2	166.0	187.2
Tainan	46741	120°18'	22°20'	12.9	523.5	195.6	205.5
Hengchun	46759	120°74'	22°01'	17.4	484.8	214.8	232.2
Hualien	46699	121°61	23°98''	12.8	465.8	210.3	217.0
Taidung	46766	121°15'	22°76'	7.8	484.0	201.4	245.2

Table 2.
Best fitting models and parameter estimates for 1910-2010 and 1960-2010.

Location	Model	Period	$\hat{a}$ (SE)	$\hat{b}$ (SE)	σ (SE)	$\hat{\xi}$ (SE)
Taipei	Gum_0	1910-2010	131.41 (5.55)		52.84 (4.12)	
	Gum_0	1960-2010	133.04 (8.39)		57.06 (6.34)	
Taichung	Gum_0	1910-2010	147.26 (7.09)		67.56 (5.32)	
	Gum_0	1960-2010	146.55 (9.58)		65.16 (7.40)	
Tainan	Gum_0	1910-2010	165.40 (7.77)		73.60 (5.48)	
	Gum_0	1960-2010	167.40 (9.81)		66.41 (7.03)	
Hengchun	GEV_0	1910-2010	196.33 (8.78)		79.6 (6.14)	-0.14 (0.06)
	Gum_1	1960-2010	151.71 (18.68)	64.85 (32.23)	69.13 (7.72)	
Hualien	GEV_1	1910-2010	157.29 (15.71)	52.21 (26.99)	76.16 (5.77)	-0.15 (0.05)
	GEV_0	1960-2010	197.55 (12.79)		80.99 (9.17)	
Taitung	Gum_1	1910-2010	144.61 (14.66)	67.01 (25.40)	69.15 (5.07)	
	Gum_1	1960-2010	166.67 (16.48)	51.34 (28.18)	61.36 (6.81)	

Table 3.
Return levels estimates for 10, 20, 50, 100-years (based on 1960-2010).

Location	10-year	20-year	50-year	100-year
Taipei	261(143,379)	302(184,420)	355(238,473)	395(277,513)
Taichung	293(175,410)	340(222,457)	400(283,518)	446(328,563)
Tainan	317(128,505)	365(175,553)	426(238,615)	472(284,661)
Hualien	343(312,391)	377(343,442)	415(374,506)	438(391,554)

Figure 1 10-year return level

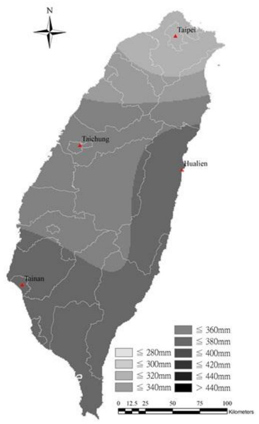

Figure 2 20-year return level

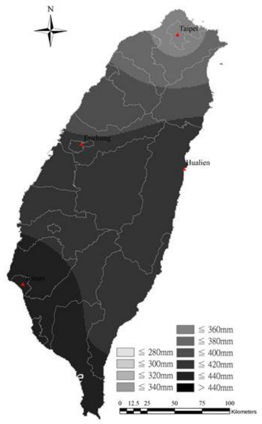

Figure 3 50-year return level